Current Events

- Financial market crash followed by sustained recession.
- Resembles history of Great Depression.
- Differs from more recent recessions in pattern.
Real S&P and Unemployment Rate, 1929-2009

- Real Price of Stock Market from Shiller
- Nonfarm, Civilian Unemployment Rate
Search and matching model, embedded within a Lucas tree economy.

No Nash bargaining to close; wage given.

Multiple equilibria chosen by confidence in the stock market.

Confidence determines wealth, which determines aggregate demand, which determines labor demand.
Extension to Farmer (2009)

- Capital not fixed.
- Introduce exogenous dissolution of matches.
- Embedded within a RBC framework.
Margins of Unemployment

- Employment varies because of
  1. Changes in participation.
  2. Changes in unemployment.

- This work will ignore participation margin.
- Normalize disutility of labor to zero.
- Focus on willing workers unable to find jobs.
- Andolfatto (1995)
- Merz (1996)
- Shimer (2005)
- Hall (2005)
- Farmer and Hollenhorst (2006)
- Hagendorn and Menovskii (2008)
- Cheremukhin and Restrepo (2008)
- Farmer (2009)
Structure of Talk

1. Solve social planner’s problem under perfect foresight.
2. Solve decentralized search economy under perfect foresight.
3. Results of numerical exercises:
   - Steady state of social planner’s problem.
   - Spectrum of equilibria under decentralized search.
   - Welfare differences between steady state equilibria.
There are measure 1 potential workers in the economy, all of whom look for jobs.

\[ 1 = L_t + U_t \]
\[ L_t = X_t + V_t \]
\[ L_t = \text{employed}, \ U_t = \text{unemployed} \]
\[ X_t = \text{production workers}, \ V_t = \text{recruiters} \]

There is a single good.

\[ Y_t = A_t K_t^\alpha X_t^{1-\alpha} \]
\[ K_t = \text{capital} \]

Capital accumulates as

\[ K_{t+1} = (1 - \delta_k) K_t + I_t \]
\[ I_t = \text{investment} \]
A labor match is generated by a matching function.

\[ m(L_t, U_t, V_t) = B_t V_t^{\eta} (U_t + \delta_l L_t)^{1-\eta} \]

\[ \delta_l = \text{exogenous separation rate} \]

The labor accumulation equation is

\[ L_{t+1} = m(L_t, U_t, V_t) + (1 - \delta_l) L_t \]
Consumers exhibit log preferences and discount the future at rate $0 < \beta < 1$.

The planner will maximize

$$\max \left\{ C_t, K_{t+1}, L_{t+1}, V_t \right\} \left\{ \sum_{t=0}^{\infty} \beta^t \ln(C_t) \right\}$$

subject to

$$C_t + K_{t+1} = A_t K_t^\alpha (L_t - V_t)^{1-\alpha} + (1 - \delta_k) K_t$$

$$L_{t+1} = B_t V_t^\eta (1 - L_t + \delta_l L_t)^{1-\eta} + (1 - \delta_l) L_t$$
Optimality Conditions

\[
\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \left( \alpha \frac{Y_{t+1}}{K_{t+1}} + 1 - \delta_k \right)
\]

\[
(1 - \alpha) \frac{Y_t}{X_t} = \Gamma_t \eta \frac{M_t}{V_t}
\]

\[
\frac{\Gamma_t}{C_t} = \frac{\beta}{C_{t+1}} \left( \left( 1 - \alpha \right) \frac{Y_{t+1}}{X_{t+1}} 
+ \Gamma_{t+1} \left( 1 - \delta_l \right) \left[ 1 - (1 - \eta) \frac{M_{t+1}}{U_{t+1}} \right] \right)
\]

where

\[
X_t = L_t - V_t
\]

\[
\Gamma_t = \text{Lagrange multiplier on labor accumulation constraint}
\]

\[
M_t = BV_t^\eta \left( U_t + \delta_l L_t \right)^{1-\eta}
\]
Decentralized Model

- The price of consumption is normalized to one.
- Households and firms do not observe the matching function.
  - Externality
The household will maximize its lifetime utility

$$\max \left\{ \sum_{t=0}^{\infty} \beta^t \ln c_t \right\} \text{ subject to }$$

$$c_t + k_{t+1} + p_t s_{t+1} + Q_{t+1} a_{t+1} = r_t k_t + w_t l_t$$

$$+ (1 - \delta_k) k_t + (p_t + d_t) s_t + a_t$$

In equilibrium, $s_t = s_{t+1} = 1$ and $a_t = a_{t+1} = 0$. 
Optimality Conditions for the Household

\[
\begin{align*}
\frac{1}{c_t} &= \frac{\beta}{c_{t+1}} (r_{t+1} + 1 - \delta_k) \\
p_t \frac{1}{c_t} &= \frac{\beta}{c_{t+1}} (p_{t+1} + d_{t+1}) \\
Q_t^{t+1} \frac{1}{c_t} &= \frac{\beta}{c_{t+1}}
\end{align*}
\]
The representative firm will want to maximize its value

\[
\max_{\{k_t, v_t\}} \left\{ p_t = \sum_{s=t}^{\infty} Q_t^s \pi_s \right\}
\]

subject to

\[
\begin{align*}
\pi_t &= A_t k_t^\alpha (l_t - v_t)^{1-\alpha} - w_t l_t - r_t k_t \\
l_{t+1} &= q_t v_t + (1 - \delta_l) l_t
\end{align*}
\]

where \( q \) is a productivity parameter measuring the effectiveness of the firm’s recruiting force.
Letting

\[ y_t = A_t k_t^\alpha (l_t - v_t)^{1-\alpha} \]

we have

\[ \alpha \frac{y_t}{k_t} = r \]

\[ \phi_t q_t = (1 - \alpha) \frac{y_t}{l_t - v_t} \]

\[ \phi_t = Q_t^{t+1} \left[ (1 - \alpha) \frac{y_{t+1}}{l_{t+1} - v_{t+1}} - w_{t+1} + \phi_{t+1} (1 - \delta_l) \right] \]

where \( \phi \) is the Lagrange multiplier of the labor accumulation constraint.
Market Clearing Conditions

- Aggregate material balance:

\[ B_t V_t^\eta (1 - L_t + \delta_l L_t)^{1-\eta} = q_t v_t \]
\[ v_t = V_t \]
\[ l_t = L_t \]

- Aggregate resource constraint:

\[ c_t + k_{t+1} = y_t + (1 - \delta_l) k_t \]
Calibration to Annual Frequency

- Standard parameterization for production function:
  \[ \alpha = 0.36 \]
  \[ \beta = 0.96 \]
  \[ \delta_k = 0.1 \]

- Shimer (2005) for matching function:
  \[ \eta = 0.5 \]
  \[ \delta_l = 0.4 \]

- Choose additional parameters to keep recruiting force small at social planner’s steady state:
  \[ A = 1 \]
  \[ B = 5 \]
Positive Productivity Shock
Steady State Values Across Different Levels of Employment

<table>
<thead>
<tr>
<th></th>
<th>Employment</th>
<th>0.99</th>
<th>0.98</th>
<th>0.95</th>
<th>0.90</th>
<th>0.85</th>
<th>0.80</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recruitment</td>
<td>0.015</td>
<td>0.015</td>
<td>0.013</td>
<td>0.011</td>
<td>0.009</td>
<td>0.008</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>1.228</td>
<td>1.216</td>
<td>1.180</td>
<td>1.120</td>
<td>1.059</td>
<td>0.998</td>
<td>0.937</td>
<td></td>
</tr>
<tr>
<td>Production</td>
<td>1.647</td>
<td>1.631</td>
<td>1.583</td>
<td>1.502</td>
<td>1.420</td>
<td>1.338</td>
<td>1.256</td>
<td></td>
</tr>
<tr>
<td>Capital Stock</td>
<td>4.185</td>
<td>4.144</td>
<td>4.022</td>
<td>3.816</td>
<td>3.609</td>
<td>3.401</td>
<td>3.192</td>
<td></td>
</tr>
<tr>
<td>Return to Capital</td>
<td>0.142</td>
<td>0.142</td>
<td>0.142</td>
<td>0.142</td>
<td>0.142</td>
<td>0.142</td>
<td>0.142</td>
<td>0.142</td>
</tr>
<tr>
<td>Wage</td>
<td>1.063</td>
<td>1.063</td>
<td>1.065</td>
<td>1.066</td>
<td>1.068</td>
<td>1.069</td>
<td>1.071</td>
<td></td>
</tr>
<tr>
<td>Price of Equity</td>
<td>0.042</td>
<td>0.040</td>
<td>0.036</td>
<td>0.030</td>
<td>0.025</td>
<td>0.021</td>
<td>0.018</td>
<td></td>
</tr>
</tbody>
</table>

- Social planner’s solution in **bold**.
The consumer is willing to give up $\lambda$ consumption in order to travel from steady state $A$ to steady state $B$.

$$U((1 + \lambda) c_A) = U(c_B)$$

Calculate how much consumption consumers are willing to give up to be at the social planner’s equilibrium, as a percentage of annual consumption:

<table>
<thead>
<tr>
<th>Employment</th>
<th>0.99</th>
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<th>0.95</th>
<th>0.90</th>
<th>0.85</th>
<th>0.80</th>
<th>0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0</td>
<td>0.01</td>
<td>0.04</td>
<td>0.09</td>
<td>0.14</td>
<td>0.19</td>
<td>0.24</td>
</tr>
</tbody>
</table>
Currently, the price of equity is calculated as

\[ p_t = Q_{t+1} (p_{t+1} + d_{t+1}) \]

where

\[ Q_{t+1} = \beta \frac{c_t}{c_{t+1}} \]

In reality, corporations own capital as well as rent it.
To match the data, must calibrate split between consumer and firm capital holdings.
Future Work

- Calculate transition path from one steady state to another.
- Calibrate division of capital stock to model stock market price-to-equity ratio.
- Drive dynamic model with shocks to price level.